Gauge Theory of Elementary Interactions

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F and D couplings of baryons and mesons are shown to arise naturally with a simple extension of the gauge formalism to a $(SU_3 \times SU_3)_L \times (SU_3 \times SU_3)_R$ group structure. In its general formulation the theory needs parity doubltets of (0+) and (0−) ninefolds. It admits of (two types of F and D coupled) (1+) and (1−) vector and axial-vector meson multiplets, a specially attractive combination of currents which emerges from the formalism being an equal mixture of vector F with axial-vector D [the case of $(SU_3)_L \times (SU_3)_R$]. The theory also admits an approximate (Bronzan-Low) type of quantum number.

1. INTRODUCTION

WITH the discovery of SU_3 as the strong interaction symmetry group,[1] with the discovery that the electromagnetic current (for strongly interacting particles) is part of the SU_3 structure,[2] and that[3] (at least in their semileptonic aspects) weak currents of strongly interacting particles also belong to SU_3, a complete theory of elementary interactions seems not too distant. In this series of papers we make a preliminary attempt towards determining their underlying group structure and to construct a gauge theory of strong, weak, and electromagnetic interactions. We believe that the gauge principle[4] must be an essential ingredient of any attempt to construct a fundamental theory. The gauge principle is the only known way to write down currents J_μ which are not simple "static" expressions of a conservation property, but also form part of the interaction Hamiltonian ($H_{\text{int}} = J_\mu A_\mu$). And on a pragmatic level, gauge theories seem to be the only spin-one theories which have so far been renormalized.[5] Our major tool is a new extension of the gauge principle to include what we call double gauges. This extension is made possible by the fact that the 'unitary' group possesses two elementary representations which admit of independent transformations. We use this new formalism to construct a theory of strong interactions in the present paper, while the problem of weak and electromagnetic interactions will be considered elsewhere.

2. THE DOUBLE GAUGE FORMALISM; LEFT AND RIGHT GAUGES

A.

We first summarize the conventional "single-gauge" formalism. Let ψ be a set of spin-$\frac{1}{2}$ particles, corresponding to an elementary (Sakata) representation of the group U_3. The single-gauge principle starts with the free-kinetic-enetgy term

$$\mathfrak{L}_f = -\psi^\dagger(\gamma_4\gamma_\mu\partial_\mu)\psi \tag{1}$$

which is invariant for the unitary transformation

$$\psi' = U_0 U \psi. \tag{2}$$

Here

$$U_0 = \exp(i\epsilon^0)\,, \quad U = \exp i(T^i\epsilon^i)\,, \tag{3}$$

T^α ($\alpha = 0, 1, \cdots, 8$) are nine Hermitian matrices which satisfy[6]

$$[T^i, T^j] = ic^{ijk}T^k\,, \quad i, j, k = 1, \cdots, 8\,, \tag{4}$$

$$\{T^i, T^j\} = (\sqrt{\tfrac{2}{3}})\delta^{ij}T^0 + d^{ijk}T^k\,, \quad i, j, k = 1, \cdots, 8\,, \tag{5}$$

$$T^0 = (1/\sqrt{6})\mathbf{1}\,, \tag{6}$$

and

$$\text{Tr}(T^\alpha)^2 = \tfrac{1}{2}.$$

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[1] The remarkable discovery of Ω^- first predicted by M. Gell-Mann [CERN Conference Report (1962)] and discovered by V. E. Barnes *et al.* [Phys. Rev. Letters **12**, 204 (1964)], seems to leave little doubt about the correctness of SU_3 symmetry. The unitary group was first introduced in elementary-particle physics by M. Ikeda, S. Ogawa, and Y. Ohnuki [Progr. Theoret. Phys. (Kyoto) **22**, 715 (1959); Y. Yamaguchi, Progr. Theoret. Phys. Suppl. (Kyoto) **11**, 37 (1959)]. These authors correctly predicted the completion of the (0−) multiplet (η,π,κ) though they followed Sakata in assigning baryons to the threefold representation. Following this the work of A. Salam and J. C. Ward [Nuovo Cimento **20**, 419 (1961)] stressed the eightfolds of both (1−) and (1+) gauge particles associated with the unitary group (the group-structure $SU_3 \times SU_3$). (The importance of spin-one multiplets lies in the fact that the gauge particles must belong to the regular representation of the symmetry group, and therefore provide the 'invariant signature' of the 'group' in contrast to any of its other representations.) The eightfold way was completed by Y. Ne'eman [Nucl. Phys. **26**, 222 (1961)], and M. Gell-Mann [Phys. Rev. **125**, 1067 (1962)], and California Institute of Technology Report CTSL 1961 (unpublished), who first pointed out that in addition to (0−) and (1−) multiplets the known baryons can also be associated with an SU_3 multiplet of eight. For some recent attempts to make use of the fundamental threefold unitary multiplet see M. Gell-Mann [Phys. Letters **8**, 214 (1964)], J. Schwinger [Phys. Rev. Letters **12**, 237 (1964)], F. Gürsey, T. D. Lee, and M. Nauenberg [Phys. Rev. **135**, B467 (1964)], and G. Zweig, Phys. Rev. (to be published).

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[6] The Greek indices run from 0 to 8, the Latin indices from 1 to 8. For definition of C^{ijk} and d^{ijk}, we use the notation of M. Gell-Mann (Ref. 1).

If we now require that the ϵ^α's in (2) depend on space-time x_μ, one must replace $\partial_\mu=\partial/\partial x_\mu$ in (1) by the 'covariant' derivative

$$\mathfrak{D}_\mu=\partial_\mu+igX_\mu+ig^0X_\mu{}^0\,, \tag{7}$$

where $X_\mu=\sqrt{2}T^iX_\mu{}^i$ and $X_\mu{}^i$ are eight vector fields with the transformation character,

$$X_\mu'=UX_\mu U^{-1}+\frac{1}{ig}U\partial_\mu U^{-1}$$

and

$$X_\mu{}^{0\prime}=X_\mu{}^0-\frac{1}{g^0}\frac{\partial\epsilon^0}{\partial x_\mu}. \tag{8}$$

The relation (8) can be inferred from the requirement

$$(\mathfrak{D}_\mu\psi)'=U(\mathfrak{D}_\mu\psi)\,. \tag{9}$$

Defining

$$\begin{aligned}X_{\mu\nu}&=\mathfrak{D}_\mu X_\nu-\mathfrak{D}_\nu X_\mu\\&=\partial_\mu X_\nu-\partial_\nu X_\mu+ig[X_\mu,X_\nu]\,,\end{aligned} \tag{10}$$

it is easy to verify that

$$X_{\mu\nu}'=UX_{\mu\nu}U^{-1}. \tag{11}$$

All in all then, the Lagrangian

$$=-\psi^\dagger\gamma_4\gamma_\mu\mathfrak{D}_\mu\psi-\tfrac{1}{4}\,\mathrm{Tr}X_{\mu\nu}X_{\mu\nu}-\tfrac{1}{4}X_{\mu\nu}{}^0X_{\mu\nu}{}^0-m\psi^\dagger\gamma_4\psi \tag{12}$$

is invariant for the transformations

$$\begin{aligned}\psi'&=U\psi\,,\\X_\mu'&=UX_\mu U^{-1}+1/igU\partial_\mu U^{-1}\,,\\X_\mu{}^{0\prime}&=X_\mu{}^0-(1/g)\partial\epsilon^0/\partial x_\mu.\end{aligned} \tag{13}$$

As usual one may define currents $J_\mu{}^\alpha$ from the relation

$$i\epsilon^\alpha J_\mu{}^\alpha=\sum_{\phi=\psi,F}(\partial\mathfrak{L}/\partial_\mu\varphi)\delta\phi\,. \tag{14}$$

From (14) and using equations of motion for φ and F

$$i\epsilon^\alpha\partial_\mu J_\mu{}^\alpha=\delta\mathfrak{L}\,, \tag{15}$$

For an invariant $\mathfrak{L}$, $\delta\mathfrak{L}=0$, so that all nine currents are conserved.

B. The Double-Gauge Formalism

For (mixed) tensor representations of the unitary group, the single-gauge formalism can be generalized in the following manner. Write the $3\times\bar{3}$ representation in the form[7]

$$\psi=\sqrt{2}(T^\alpha\psi^\alpha)\,. \tag{16}$$

[7] To identify the transformation character of the fields ψ^α, consider the corresponding (Hermitian) boson matrix $M=\sqrt{2}M^\alpha T^\alpha$. With the notation

$$\pi^\pm=(1/\sqrt{2})(M^1\mp iM^2),\quad K^\pm=(1/\sqrt{2})(M^4\mp iM^5)$$

$$K^0,\bar{K}^0=(1/\sqrt{2})(M^6\mp iM^7),\quad \pi^0=M^3,\quad \eta^0=M^8,\quad \sigma^0=M^0$$

$$M=\begin{pmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}}+\frac{\sigma^0}{\sqrt{3}} & \pi^+ & K^+\\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}}+\frac{\sigma^0}{\sqrt{3}} & K^0\\ K^- & \bar{K}^0 & \frac{-2\eta^0}{\sqrt{6}}+\frac{\sigma^0}{\sqrt{3}}\end{pmatrix}.$$

The single-gauge formalism would start in this case with the transformation

$$\psi=U_0U\psi U^{-1}. \tag{17}$$

We generalize this to consider two *independent* unitary transformations U_1 and U_2,

$$\psi'=U_0U_1\psi U_2{}^{-1} \tag{18}$$

which leave $\mathfrak{L}_f$ invariant,

$$\mathfrak{L}_f=-\mathrm{Tr}\psi^\dagger\gamma_4\gamma_\mu\partial_\mu 4-m_0\,\mathrm{Tr}\psi^\dagger\gamma_4\psi.$$

The 'gauge principle' leads in this case to the covariant derivative,

$$\mathfrak{D}_\mu\psi=\partial_\mu\psi+ig_1'Z_{1\mu}\psi-ig_2'\psi Z_{2\mu}+ig'^0Z_\mu{}^0\psi\,, \tag{19}$$

which transforms as

$$(\mathfrak{D}_\mu\psi)'=U_0U_1(\mathfrak{D}_\mu\psi)U_2{}^{-1}\,, \tag{20}$$

provided

$$(Z_{1\mu})'=U_1(Z_{1\mu})U_1{}^{-1}+(1/ig_1')U_1\partial_\mu U_1{}^{-1}\,, \tag{21}$$

$$(Z_{2\mu})'=U_2(Z_{2\mu})U_2{}^{-1}+(1/ig_2')U_2\partial_\mu U_2{}^{-1}\,, \tag{22}$$

$$Z_\mu{}^{0\prime}=Z_\mu{}^0-(1/g^{0\prime})\partial\epsilon^0/\partial x_\mu.$$

The crucial remark is that each of the fields Z_1 and Z_2 transforms *independently* as representation[8] of SU_3. The invariant Lagrangian is given by

$$\mathfrak{L}=-\mathrm{Tr}\psi^\dagger\gamma_4\gamma_\mu\mathfrak{D}_\mu\psi-\tfrac{1}{4}\,\mathrm{Tr}\Sigma Z_{\mu\nu}Z_{\mu\nu}-\mathrm{Tr}m_0\psi^\dagger\gamma_4\psi. \tag{23}$$

The fermion interaction term in (23) equals

$$\mathfrak{L}_{\mathrm{int}}\approx\mathrm{Tr}[-i\psi^\dagger\gamma_4\gamma_\mu(g_1'Z_{1\mu}\psi-g_2'\psi Z_{2\mu})-ig^0\psi^\dagger\gamma_4\gamma_\mu\psi Z_\mu{}^0] \tag{24}$$

$$=\mathrm{Tr}[-i/\sqrt{2}(\psi^\dagger\gamma_4\gamma_\mu[F_\mu,\psi]+\psi^\dagger\gamma_4\gamma_\mu\{D_\mu,\psi\})-ig^0\psi^\dagger\gamma_4\gamma_\mu\psi Z_\mu{}^0], \tag{25}$$

where

$$F_\mu=T^iF_\mu{}^i=1/\sqrt{2}(g_1'Z_{1\mu}+g_2'Z_{2\mu})\,,$$

$$D_\mu=T^iD_\mu{}^i=1/\sqrt{2}(g_1'Z_{1\mu}-g_2'Z_{2\mu})\,.$$

There is thus a total of 18 conserved currents, corresponding to the group generators, eight grouped in the commutator combination,

$$\mathrm{Tr}\psi^\dagger O[F,\psi]=\mathrm{Tr}F(-\psi O^i\psi^\dagger-\psi^\dagger O\psi)\,, \tag{26}$$

and the remaining in the anticommutator,

$$\mathrm{Tr}\psi^\dagger O\{D\psi\}=\mathrm{Tr}D(-\psi O^i\psi^\dagger+\psi^\dagger O\psi)=\sqrt{2}d^{ijk}(\psi^{\dagger i}O\psi^j)D^k+\sqrt{2}(\psi^{\dagger j}O\psi^0+\psi^{0\dagger}O\psi^j)D^j. \tag{27}$$

The currents remain conserved even for the addition of gauge-meson mass terms to (23)

$$\mathfrak{L}_m=-\tfrac{1}{2}\,\mathrm{Tr}(\mu_1{}^2Z_{1\mu}Z_{1\mu}+\mu_2{}^2Z_{2\mu}Z_{2\mu})\,. \tag{28}$$

[8] Terms like (28) are not invariant for the general transformation (21) and (22). They are invariant however for non x_μ-dependent U_1's and U_2's. This is all that is necessary for current conservation.

C. R Parity

D and F interactions can be distinguished if following Gell-Mann one defines R parity η for a field from the relation

$$R\psi R^{-1}=\eta_\psi\psi^T \quad \eta_\psi=\pm 1\,.$$

Since

$$\mathrm{Tr}A[B,C]=-\mathrm{Tr}A^T[B^T,C^T]$$
$$\mathrm{Tr}A\{B,C\}=+\mathrm{Tr}A^T\{B^T,C^T\}\,,$$

D and F transform oppositely. If the theory is R invariant, one must choose

(1) $g_1'=g_2', \quad \mu_1=\mu_2$
(in order that D_μ and F_μ are orthogonal).[9]

(2) $\eta_D=+1, \quad \eta_F=-1\,.$

3. INCLUSION OF AXIAL VECTOR INTERACTIONS

A.

Neglecting the fermion mass term $-m_0\,\mathrm{Tr}\psi^\dagger\gamma_4\psi$, one may split ψ into its right and left components$_\mu$

$$\psi_{L,R}=\tfrac{1}{2}(1\pm\gamma_5)\psi\,. \tag{29}$$

As before consider the independent transformations

$$\begin{aligned}\psi_R'&=V_0V_1\psi_RV_2^{-1}\\ \psi_L'&=V_0'V_3\psi_LV_4^{-1}\,.\end{aligned} \tag{30}$$

To gauge the free Lagrangian

$$-\mathrm{Tr}(\psi_R{}^\dagger\gamma_4\gamma_\mu\partial_{\mu R}\psi_R+\psi_L{}^\dagger\gamma_4\gamma_\mu\partial_{\mu L}\psi_L)\,,$$

replace

$$\begin{aligned}\partial_{\mu R}\ \text{ by }\ \mathfrak{D}_{\mu R}\psi_R&=\partial_\mu\psi_R+ig_1F_{1\mu}\psi_R\\ &\quad-ig_2\psi_RF_{2\mu}+ig_0F_0\psi_R\,,\\ \partial_{\mu L}\ \text{ by }\ \mathfrak{D}_{\mu L}\psi_L&=\partial_\mu\psi_L+ig_3F_{3\mu}\psi_L\\ &\quad-ig_4\psi_LF_{4\mu}+ig_0'F_0'\psi_L\,.\end{aligned} \tag{31}$$

The fields F_1, F_2, F_3, F_4 transform independently of each other. Incorporating the g's in the definition of the F's, the linear part of $\mathfrak{L}_{\mathrm{int}}$ equals

$$\begin{aligned}\mathfrak{L}_{\mathrm{int}}=&-i\,\mathrm{Tr}(\psi_R{}^\dagger F_1\psi_R-\psi_R{}^\dagger\psi_RF_2+\psi_L{}^\dagger F_3\psi_L\\ &\qquad-\psi_L{}^\dagger\psi_LF_4+\psi_R{}^\dagger\psi_RF_0+\psi_L{}^\dagger\psi_LF_0')\\ =&-\tfrac{1}{4}i\,\mathrm{Tr}\psi^\dagger\gamma_4\gamma_\mu\{F_1+F_3-F_2-F_4,\psi\}\\ &+\psi^\dagger\gamma_4\gamma_\mu[F_1+F_3+F_2+F_4,\psi]\\ &+\psi^\dagger\gamma_4\gamma_\mu\gamma_5\{-F_1+F_3+F_2-F_4,\psi\}\\ &+\psi^\dagger\gamma_4\gamma_\mu\gamma_5[-F_1+F_3-F_2+F_4,\psi]\\ &+\tfrac{1}{2}i\psi^\dagger\gamma_4\gamma_\mu\psi(F_0+F_0')\\ &+\tfrac{1}{2}i\psi^\dagger\gamma_4\gamma_\mu\gamma_5(F_0-F_0')\,.\end{aligned} \tag{32}$$

There are four types of conserved currents $D-V$, $D-A$, $F-V$, $F-A$ corresponding to the 36 generators for the underlying group structure. We show in Appendix I that the vector currents remain conserved even for the inclusion of Fermi mass terms.

B. Spin-Zero Fields

For Hermitian spin-zero fields M, the only transformation which preserves hermiticity is the single-gauge transformation

$$M'=UMU^{-1}\,.$$

For a non-Hermitian M, $M=(M_1+iM_2)$ one can however allow double gauges

$$M'=U_0U_1MU_2^{-1}\,.$$

Infinitesimally, $U_1\approx 1+iX_1$, $U_2\approx 1+iX_2$,

$$\begin{aligned}M_1'&=M_1-\tfrac{1}{2}\{X_1-X_2,M_2\}+\tfrac{1}{2}i[X_1+X_2,M_1]-\epsilon^0M_2\,,\\ M_2'&=M_2+\tfrac{1}{2}\{X_1-X_2,M_1\}+\tfrac{1}{2}i[X_1+X_2,M_2]+\epsilon^0M_1\,.\end{aligned}$$

The invariant Lagrangian equals

$$\begin{aligned}&-\tfrac{1}{2}\,\mathrm{Tr}(\mathfrak{D}_\mu M)^+(\mathfrak{D}_\mu M)-\tfrac{1}{2}\,\mathrm{Tr}\kappa^2M^+M\\ &\quad=-\tfrac{1}{2}\,\mathrm{Tr}\begin{pmatrix}(\partial_\mu M_1+i/\sqrt{2}[V_\mu,M_1]-1/\sqrt{2}\{A_\mu,M_2\})^2\\ +(\partial_\mu M_2+i/\sqrt{2}[V_\mu,M_2]+1/\sqrt{2}\{A_\mu,M_1\})^2\end{pmatrix}\\ &\qquad-\tfrac{1}{2}\kappa^2\,\mathrm{Tr}(M_1^2+M_2^2)\,,\end{aligned} \tag{33}$$

where

$$V_\mu=1/\sqrt{2}(X_1+X_2)_\mu,\quad A_\mu=1/\sqrt{2}(X_1-X_2)_\mu\,. \tag{34}$$

The conserved currents are

$$\begin{aligned}\frac{\delta\mathfrak{L}_{\mathrm{meson}}}{\delta V_\mu}&=\frac{-i}{\sqrt{2}}([\partial_\mu M_1,M_1]+[\partial_\mu M_2,M_2])\,,\\ \frac{\delta\mathfrak{L}_{\mathrm{meson}}}{\delta A_\mu}&=\frac{1}{\sqrt{2}}(\{\partial_\mu M_1,M_2\}-\{\partial_\mu M_2,M_1\})\,.\end{aligned} \tag{35}$$

4. TOWARDS A THEORY OF STRONG INTERACTIONS

At this stage with one fermion nine-fold$=\psi_R+\psi_L$ corresponding to the group structure $(\mathrm{SU}_3\times\mathrm{SU}_3)_L\times(\mathrm{SU}_3\times\mathrm{SU}_3)_R$ and one meson ninefold $M=M_1+iM_2$, each gauged independently as

$$\begin{aligned}\psi_R'&=V_0V_1\psi_RV_2^{-1}\,,\\ \psi_L'&=V_0'V_3\psi_LV_4^{-1}\,,\\ M'&=U_0U_1MU_2^{-1}\,,\end{aligned} \tag{36}$$

there is a total of six types of gauge fields; four of these, i.e., F_1, F_2, F_3, F_4 interact only with fermions ψ and two fields X_1 and X_2 only with mesons M. In order that fermions and mesons interact with each other at all some of the U transformations must be identified with the V transformations subject of course to P and C conservation.

In Appendix II we list P, C, and R parities of the currents in (32) and (33). This listing shows also the resistrictive power of the gauge formalism which

[9] Note that in terms of the 'pure' fields F and D,

$$Z_{1\mu\nu}Z_{1\mu\nu}+Z_{2\mu\nu}Z_{2\mu\nu}=(F_{\mu\nu}F_{\mu\nu}+D_{\mu\nu}D_{\mu\nu}),$$

where

$$\begin{aligned}F_{\mu\nu}&=\partial_\mu F_\nu-\partial_\nu F_\mu+ig[F_\mu,F_\nu]+ig[D_\mu,D_\nu],\\ D_{\mu\nu}&=\partial_\mu D_\nu-\partial_\nu D_\mu+ig[D_\mu,F_\nu]+ig[F_\mu,D_\nu].\end{aligned}$$

stems from the fact that "pure" P- and R-conjugate fields are 'mixtures' of the gauge F's and X's. (R-invariance has been listed in so far as it may prove an approximate symmetry for strong interactions).

To construct a realistic theory, consider the case where we start with one scalar and one pseudoscalar boson multiplet:

$$M_1=0-\,,\quad M_2=0+\,.$$

A. P Invariance

From (A1) and (A5) we must require for parity conservation:

$$g_1'=g_3'\,,\quad g_2'=g_4'\,,$$

and either

$$X_1=F_1\,,\quad X_2=F_3\,,$$

i.e.,

$$U_1\equiv V_1\,,\quad U_2\equiv V_3\,,$$

or

$$X_1=F_2\,,\quad X_2=F_4\,,$$

i.e.,

$$U_1\equiv V_2\,,\quad U_2\equiv V_4\,.$$

Using the notation of (34), i.e.,

$$\sqrt{2}V=F_1+F_3=X_1+X_2\,,\quad \sqrt{2}V'=F_2+F_4,$$
$$\sqrt{2}A=F_1+F_3=X_1-X_2\,,\quad \sqrt{2}A'=F_2-F_4,$$

the linear part of the $\mathfrak{L}_{\rm int}$ equals:

$$\begin{aligned}&[\gamma_\mu]S^V(F)+\{\gamma_\mu\}S^V(D)+[\gamma_\mu\gamma_5]S^A(F)+\{\gamma_\mu\gamma_5\}S^A(D)\\ &\quad+[M_1,\partial M_1][S^V(F)+S^V(D)]\\ &\quad+[M_1,\partial M_2][S^A(F)+S^A(D)]\\ &\quad+A^0\,\mathrm{Tr}(\psi^\dagger\gamma_4\gamma_\mu\gamma_5\psi)+(M_1\partial_\mu M_2-M_2\partial_\mu M_1)\,.\end{aligned}\tag{37}$$

B. γ_5 Invariance

The P-invariant gauge Lagrangian (37) is invariant also for the "γ_5" transformation

$$\begin{aligned}&\psi_R\to\psi_L\\ &\left.\begin{aligned}F_1\to F_3\\ F_2\to F_4\end{aligned}\right\}\ \text{i.e.,}\ \left.\begin{aligned}V,V'\to+(V,V')\\ A,A'\to-(A,A')\end{aligned}\right\}\\ &M_1\to -M_1\\ &M_2\to +M_2\,.\end{aligned}$$

[To see this most simply refer to (31) and (33).]

Since $\psi_R\to\psi_L$ is a tenable transformation, only if mass terms vanish, we have the important result that *if fermion masses are ignored in fermion loops, the following quantum number is respected for all processes involving external bosons:*

$$\begin{aligned}+1\quad&\text{for}\quad V(1-)\quad\text{and}\quad M_2(0+)\,,\\ -1\quad&\text{for}\quad A(1+)\quad\text{and}\quad M_1(0-)\,.\end{aligned}\tag{38}$$

In Sec. 5 we shall discuss the relation of this to the quantum number recently introduced by Bronsan and Low.

C. R Invariance

The Lagrangian (37) is not R invariant. This is because from (A.3) $\mathfrak{L}_{\rm fermi}$ is RP invariant only if

$$\begin{aligned}&g=g'\,,\\ &RP\,V(RP)^{-1}=V'^T\,,\\ &RP\,A(RP)^{-1}=A'^T\,,\end{aligned}\tag{39}$$

while from $\mathfrak{L}_{\rm meson}$ is invariant only if

$$\begin{aligned}&RP\,V(RP)^{-1}=V^T\,,\\ &RP\,A(RP)^{-1}=-A^T\,.\end{aligned}\tag{40}$$

These clearly are contradictory requirements, and therefore (37) is R invariant if and only if

$$\begin{aligned}&V=V'\qquad (X_1=F_1=F_4)\,,\\ &A=-A'\quad (X_2=F_2=F_3)\,.\end{aligned}\tag{41}$$

D. The Structure $(\mathrm{SU}_3)_L\times(\mathrm{SU}_3)_R$

The choice (41) is highly restrictive. Explicitly, the relevant gauge transformations are

$$\begin{aligned}&\psi_R'=U_0V_1\psi_RV_2^{-1}\,,\\ &\psi_L'=U_0,V_2\psi_LV_1^{-1}\,,\\ &M'=V_1MV_2^{-1}\,.\end{aligned}\tag{42}$$

There are just two gauge fields $S^V(F)$ and $S^A(D)$ and the (linear part of) $\mathfrak{L}_{\rm int}$ equals

$$\begin{aligned}&S^V(F)([\gamma_\mu]+[\partial M_1,M_1]+[\partial M_2,M_2])\\ &\quad+S^A(D)(\{\gamma_\mu\gamma_5\}+\{\partial M_2,M_1\}-\{\partial M_1,M_2\})\,.\end{aligned}\tag{43}$$

This remarkable mixture of equal parts of $(V-F)+(A-D)$ currents seems to present the most attractive choice for a first approximation to a strong interaction theory. We discuss this further in Sec. 5.

Note that the transformations (42) leave Yukawa-like terms

$$i\,\mathrm{Tr}(\psi_R^\dagger\gamma_4\gamma_5M\psi_L+\mathrm{H.c.})=i\,\mathrm{Tr}\psi^\dagger\gamma_4\gamma_5M_1\psi-\mathrm{Tr}\psi^\dagger\gamma_4M_2\psi$$

invariant. These in fact are the only 'double' transformations to do so.

E. Other Special Cases

It is not of course, essential to assume that (as we did so far) that M_1 and M_2 are particles of opposite parities. If there exist two basic pseudoscalar ninefolds (M_1, $M_2=0-$), a PR-invariant Lagrangian can be constructed as follows:

(1) *For even* (M_1,M_2) *R-parity*, if

$$\begin{aligned}&X_1=F_1=F_3\,,\\ &X_2=F_2=F_4\,.\end{aligned}$$

No A couplings, though both $V-F$ and $V-D$ interactions exist.

(2) *For odd* (M_1, M_2) *R-parity*, if

$$X_1 = X_2 = F_1 = F_2 = F_3 = F_4$$

only $V-F$ couplings are allowed.

5. FROM $(SU_3)_L \times (SU_3)_R$ TO SU_3

As shown in (37) a parity-conserving strong interaction theory admits of four sets of gauge particles $S^V(F)$, $S^V(D)$, $S^A(F)$, and $S^A(D)$, with spin-zero mesons interacting equally with $S(F)+S(D)$ or $S(F)-S(D)$ combinations. If R invariance is further imposed, we get an economical special choice which exhibits a 'symmetry' between $V-F$ and $A-D$ couplings. For either theory to represent the physical situation we need (1) the existence of a ninefold $M_1(0-) = (\eta, \pi, \kappa, \sigma)$ and a ninefold $M_2(0+) = (\eta', \pi', \kappa', \sigma')$, besides the fermion ninefold ψ (possibly Λ, Σ, Ξ, N, Y_0^*); and (2) the existence of both $(1-)$ and $(1+)$ gauge particles.

Consider for simplicity the special theory given by the Lagrangian in (43) [though all considerations apply equally to the general case (37)]. Without the Fermi mass term, (43) is invariant for $(U_3 \times U_3)_R \times (U_3 \times U_3)_L$ transformations (42). For strong interactions we wish however to particularize to SU_3, i.e., to invariance for $A' = XAX^{-1}$, $X = \exp i(T^i \epsilon^i)$ $i = 1, \cdots 8$.

There are two equivalent ways of achieving this.

(a) Introduce the SU_3 [but not $(SU_3)_L \times (SU_3)_R$ invariant] fermion mass term

$$\mathfrak{L}_m = -(m_0) \operatorname{Tr} \psi^\dagger \gamma_4 \psi . \qquad (44)$$

Using Appendix I,

$$\partial_\mu J_\mu(V) = 0 ,$$
$$\partial_\mu J_\mu\langle(A) = -2m_0 \quad \psi^\dagger \gamma_4 \gamma_5 \{\psi, T\alpha\} .$$

(b) One may rewrite (44) in the form

$$\mathfrak{L}_m = i \operatorname{Tr} \psi_R^\dagger \gamma_4 \gamma_5 \langle M \rangle \psi_L + \text{H.c.} ,$$

where we assume that the expectation value M is nonzero only for the component M_2^0; i.e.,

$$\langle \sigma' \rangle = \langle M_2^0 \rangle = \sqrt{3} m_0 . \qquad (45)$$

One may now redefine the field M_2, in the following manner

$$M_2 = M_2' + \sqrt{3} m_0 .$$

In terms of M_2', (43) equals

$$\mathfrak{L}_{(M)} = \mathfrak{L}_{(M')} - (3m_0^2 g^2) \operatorname{Tr}(S^A)^2 + m_0/2 \operatorname{Tr}(\partial M_1 S^A) - m_0/\sqrt{2} \operatorname{Tr}(\{S^A, M_2'\} S^A) . \quad (46)$$

The extra (SU_3 invariant) terms on the right porvide a resolution[10] of the standard dilemma of gauge theories—the inability to provide for single emission of Yukawa mesons. This is because the interaction term $S^A \partial M_1$ combined with $\psi^\dagger \psi S^A$, gives an effective Yukawa *pseudovector D* interaction proportional to

$$m_0 \psi^\dagger(x_1) \gamma_4 \gamma_\mu \gamma_5 \{ (\partial M_1 / \partial X_\nu)(x_2), \psi(x_1) \} \Delta_{F\mu\nu}(x_1 - x_2) .$$

Note that the ansatz (45) gives an additional mass term in (46) for the S_A particles, in contrast to the S_V particles.

(c) *Summarizing*, the (ill-understood) vacuum-degeneracy assumption $\langle M_2^0 \rangle \neq 0$ gives a natural way to reduce the general symmetry to SU_3. This mechanism seems to be connected with the appearance of fermion mass [see (b) above], with the Yukawa coupling constant and with a mass difference between the $(1+)$ and $(1-)$ gauge particles.

(d) The special $(V-F)+(A-D)$ theory admits of PR invariance for the full Lagrangian with PR parities:

$$+1 \text{ for } \rho, \phi, K^* \text{ and } \sigma', \pi', \eta', k' ,$$
$$-1 \text{ for } \omega(=S_0^V) \text{ and } \sigma, \pi, \eta, k .$$

For the general non-PR-invariant Lagrangian (37) naturally no such assignment is possible. However, as remarked earlier, this general case too admits of the quantum number:

$+1$ for $(1-)$ particles like ω, ρ, ϕ, K^* and for $(0+)\sigma'$, π', η', K',

-1 for $(1+)$ particles like ω', ρ', ϕ', $K^{*\prime}$ and for $(0-)\sigma$, π, η, K,

when the fermion mass term is ignored. This quantum number agrees with the Bronzan-Low number A (conserved only when fermion interactions are altogether omitted) for γ, ρ, ϕ and π, η, K. For ω there is no agreement and therefore in our scheme $\omega \to 3\pi$ is forbidden in the limit of fermion masses zero.

(e) Assuming further that $\langle \eta' \rangle = \langle M_2^8 \rangle \neq 0$ Sakurai, Glashow, and Coleman[11] have succeeded in giving a coherent picture of SU_3 breaking. Clearly even if this assumption is superposed on the theory, the R invariance of the special symmetrical Lagrangian (43) will not allow any $p-\Xi$ mass difference to develop. This special $(V-F)+(A-D)$ theory can therefore only be an approximation to the physical situation a representation for which is provided by the non-PR-invariant expression (37) with,

$$\sigma = \sigma' + \sqrt{3} m_0 ,$$
$$\eta = \eta' + \langle \eta_0' \rangle .$$

Concluding, the double-gauge principle leads in a natural manner to D as well as F interactions. We may go further and in fact assert that double gauges are *necessary* for the gauge appearance of D currents in the interaction Lagrangian. Further in order that both

[10] A. Salam and J. C. Ward, Nuovo Cimento **19**, 167 (1961).

[11] The tadpole mechanism was introduced in elementary-particle physics by J. Schwinger, Ann. Phys. (N.Y.) **2**, 407 (1957). It was used by A. Salam and J. C. Ward (Ref. 10) for strong and weak interactions. For applications to derive mass formulas see J. J. Sakurai [Phys. Rev. **132**, 434 (1963)]; and S. Coleman and S. Glashow, Harvard (to be published).

V and A gauge couplings can exist, the theory *must* be parity-doublet symmetric, i.e., both (1+) and (1−) as well as (0+) and (0−) particles have to appear. The underlying group structure is not simply the structure of SU_3 but corresponds in general to $(SU_3 \times SU_3)_L \times (SU_3 \times SU_3)_R$.

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APPENDIX I

Write $\mathcal{L}_{\text{mass}} = m_0 \operatorname{Tr}(\psi_R^\dagger \gamma_4 \psi_L + \psi_L^\dagger \gamma_4 \psi_R)$. For the general transformation (25)

$$\mathcal{L}_{\text{mass}}' = m_0 \operatorname{Tr}(\psi_R^\dagger \gamma_4 R \psi_L S + \text{H.c.}),$$

where $R = V_1^{-1} V_3$, $S = V_4^{-1} V_2$. Infinitesimally [for notation see (3)]

$$\delta\mathcal{L} = i m_0 (\psi_R^\dagger \gamma_4 R^\alpha (\epsilon_3 - \epsilon_1)^\alpha \psi_L + \psi_R^\dagger \gamma_4 \psi_L T^\alpha (\epsilon_2 - \epsilon_4)^\alpha - \psi_L^\dagger \gamma_4 T^\alpha (\epsilon_3 - \epsilon_1)^\alpha \psi_R - \psi_L^\dagger \gamma_4 \psi_R T^\alpha (\epsilon_2 - \epsilon_4)^\alpha .$$

Using (15)

$$\partial_\mu J_\mu (F_1 + F_3) = 0 = \partial_\mu J_\mu (F_2 + F_4).$$

This shows that the vector currents are always conserved. Also

$$\partial_\mu J_\mu^\alpha (F_3 - F_1) = m_0 \psi^\dagger T^\alpha (\gamma_4 \gamma_5) \psi ,$$
$$\partial_\mu J_\mu^\alpha (F_2 - F_4) = m_0 \psi^\dagger (\gamma_4 \gamma_5) \psi T^\alpha .$$

Thus

$$\partial_\mu J_\mu^\alpha (A - D) \alpha m_0 \psi^\dagger \gamma_4 \gamma_5 \{T^\alpha, \psi\}$$
$$\partial_\mu J_\mu^\alpha (A - F) \alpha m_0 \psi^\dagger \gamma_4 \gamma_5 [T^\alpha, \psi].$$

APPENDIX II

A. Bilinear Currents

Let

$$F = \psi O^t \psi^\dagger + \psi^\dagger O \psi ,$$
$$D = \psi O^t \psi^\dagger - \psi^\dagger O \psi ,$$

where

$$O = \gamma_4 \gamma_\mu = O^t \quad (V),$$
$$O = \gamma_4 \gamma_\mu \gamma_5 = -O^t \quad (A),$$
$$O = \gamma_4 \gamma_5 = -O^t \quad (P),$$
$$O = \gamma_4 = -O^t \quad (S),$$

and O^t are the transposed γ's (in the Majorana representation). Note, for a Hermitian field F, $(RC)F(RC)^{-1} = \pm F$.

Consider the Fermi Lagrangian (32).

(1) *For P invariance we need*

$$P\ F_1 P^{-1} = -F_3 ,\qquad P\ F_2 P^{-1} = -F_4 , \tag{A1}$$

and

$$g_1' = g_3' ,\quad m_{F_1} = m_{F_3} ,$$
$$g_2' = g_4' ,\quad m_{F_2} = m_{F_4} .$$

(2) *For R invariance:*

$$R\ F_1 R^{-1} = -F_2^T ,\qquad R\ F_3 R^{-1} = -F_4^T , \tag{A2}$$

and

$$g_1' = g_2' ,\quad g_3' = g_4' .$$

(3) *For RP invariance:*

$$PR\ F_1 (PR)^{-1} = +F_4^T ,\qquad PR\ F_2 (PR)^{-1} = +F_3^T . \tag{A3}$$

(4) *For RC invariance:*

$$CR\ F_1 (CR)^{-1} = F_4 ,\qquad CR\ F_2 (CR)^{-1} = F_3 . \tag{A4}$$

TABLE I. Table of parities.

		R	RC	RP	RPC
V	F/D	−1 +1	+1 −1	+1 −1	−1 +1
A	F/D	−1 +1	−1 +1	−1 +1	−1 +1
P	F/D	−1 +1	−1 +1	+1 −1	+1 −1
S	F/D	−1 +1	−1 +1	−1 +1	−1 +1

B. Meson Fields

Depending on relative R and P parities of M_1, M_2, the following cases arise for (33):

(1) *For P invariance we need:*

$$PX_1P^{-1} = -X_2 \text{ if } M_1 = 0-,\ M_2 = 0+,$$
$$PX_1P^{-1} = -X_1 \text{ for even relative } M_1, M_2\ P \text{ parity.} \tag{A5}$$

(2) *R invariance:*

$$RX_1R^{-1} = -X_2^T \text{ for even } (M_1, M_2)\ R \text{ parity}$$
$$= -X_1^T \text{ for odd } R \text{ parity.} \tag{A6}$$

(3) *RP invariance:*

$$RPX_1(RP)^{-1} = X_1^T \text{ for odd } P \text{ and even } R \text{ parity}$$
$$= X_2^T \text{ for odd } P \text{ and odd } R \text{ parity.} \tag{A7}$$

(4) *RC invariance:*

$$(RC)X_1(RC)^{-1} = X_1 \text{ for even } R \text{ parity}\quad (RC)M(RC)^{-1} = M$$
$$(RC)X_1(RC)^{-1} = +X_2 \text{ for odd } R \text{ parity}\quad (RC)M(RC)^{-1} = M^+ . \tag{A8}$$